Rugate and discrete hybrid filter designs

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Abstract

The combination of discrete and Rugate filter design techniques can provide unique design advantages. Rugate filter deposition techniques can be applied to discrete, and square wave based designs as easily as they are applied to sine wave index profiles. Just as apodization reduces sidelobes about a Rugate stop-band, apodization reduces sidelobes of a square wave filter. This paper builds on these observations to present designs which superimpose sine and square wave profiles to produce a more efficient use of the design space. Techniques are presented for moving back and forth between discrete and Rugate designs to achieve reduced film thickness, more efficient use of index contrast, and harmonic suppression.

Keywords: optical filters, Rugates, thin film design

1.0 Introduction

Optical interference films are used in most optical systems to control or enhance spectral performance. These films are thin layers or blends of optical materials of different refractive index. When light passes through a change in refractive index, partial reflection occurs. The coherence of these subtle reflections determines the nature of the filter's optical spectrum[1]. Two means of designing and fabricating interference filters are discrete stacks and Rugates. Discrete stack filters are alternating layers of optical material. Rugate filters are a continuously graded, periodic blend of two optical materials. The mixing ratio of the material blend determines the immediate refractive index of the film. Co-deposition makes Rugate filters more challenging to fabricate than discrete stacks, however they offer a number of design and performance advantages.

Figure sets 1 and 2 present a comparison of a discrete stack filter and a Rugate designed to similar parameters with the exception of wave form. Table 1 lists the design parameters for these example designs. Index profiles are presented in figures 1a and 2a, and predicted performance is presented in figures 1b and 2b. The quarter wave stack filter exhibits a primary harmonic at the design wavelength of 2 microns, and a series of odd ordered harmonics. The presents of the high ordered harmonics is suggested from the Fourier transform of the square wave index profile[2]:

$$n = a*\sin(OT/\lambda) + a/3*\sin(3*OT/\lambda) + a/5*\sin(5*OT/\lambda) + a/7*\sin(7*OT/\lambda) + \ldots$$

where: a = one half the index contrast between materials;
OT = optical thickness; and
λ = the wavelength of the primary harmonic.

The Rugate notch filter design does not exhibit any significant high ordered harmonics. A weak, second harmonic which appears in large index contrast, or high optical density designs, such as in this example, can be completely suppressed by using a slight variation of the sine wave index profile[3] - the exponential sine. Figure set 3 presents the example design using this functional form. Figure set 4 explores the difference in wave form a little more closely. The sine and exponential sine index profiles are overlaid in figure 4a, and the difference between the two are plotted in figure 4b. The difference in the profiles approximates a weak sine wave profile at twice the primary frequency. For moderate bandwidth and optical density designs, the difference between sine and exponential waveforms is not significant. Figure set 5 overlays sine and exponential sine index and modeled performance for a 0.1 amplitude, optical density 3 notch filter design.

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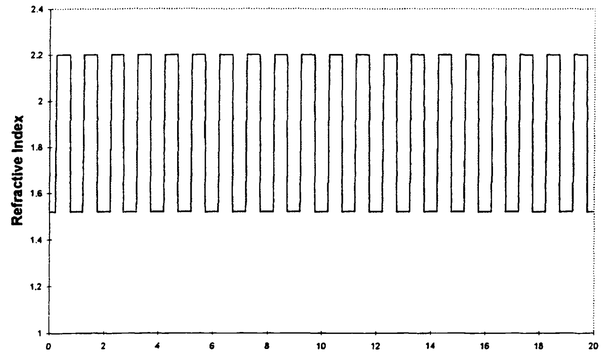

Figure 1a: Discrete index profile for a 2 micron quarter wave notch filter. The index contrast is what might be expected for a TiO2/SiO2 film system.

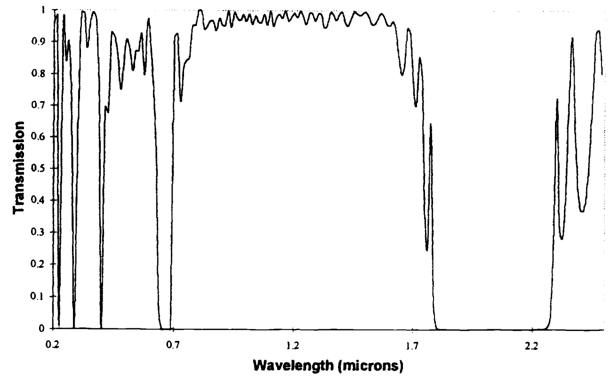

Figure 1b: Predicted performance for the discrete, quarter wave stack filter. The odd ordered harmonics at 0.66, 0.4, 0.28, and 0.22 microns are characteristic of discrete filters.

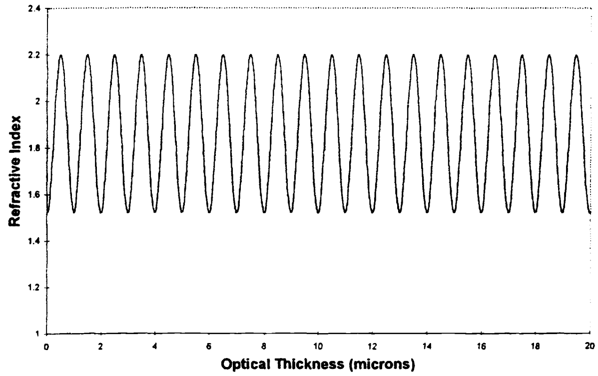

Figure 2a: Rugate sine wave index profile for a 2 micron Rugate notch filter. This filter is designed to the same index contrast as figure set 1.

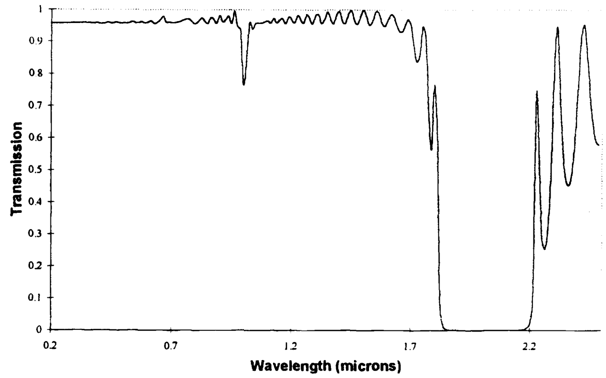

Figure 2b: Predicted performance for the Rugate, sine wave gradient index filter. The sine wave index profile exhibits good high ordered harmonic suppression, however, a weak, second order harmonic is present.

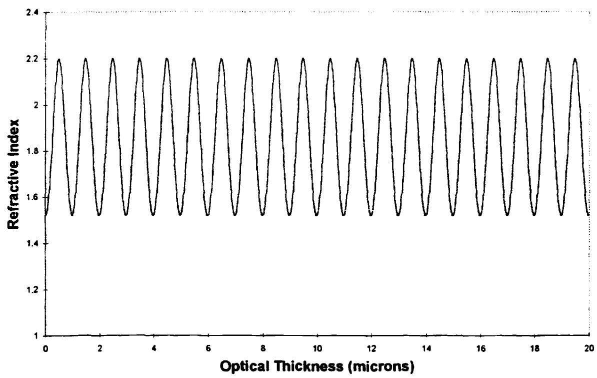

Figure 3a: Rugate exponential sine index profile for a 2 micron Rugate notch filter. This filter is designed to the same index contrast as figure set 1.

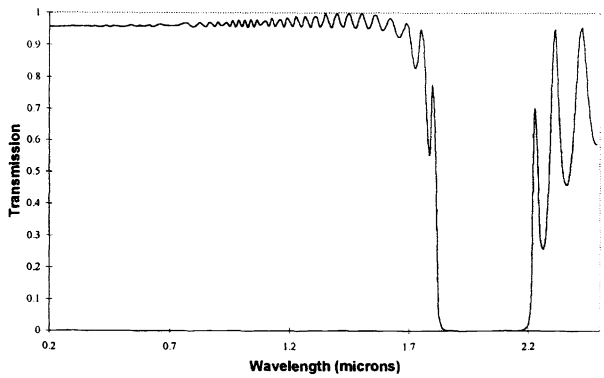

Figure 3b: Predicted performance for the Rugate, exponential sine gradient index filter. This waveform gives very good high order harmonic suppression.

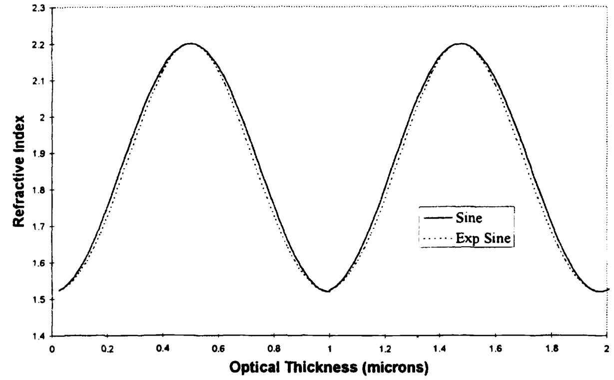

Figure 4a: Sine and exponential sine index profiles are overlaid illustrating the subtle differences in waveforms.

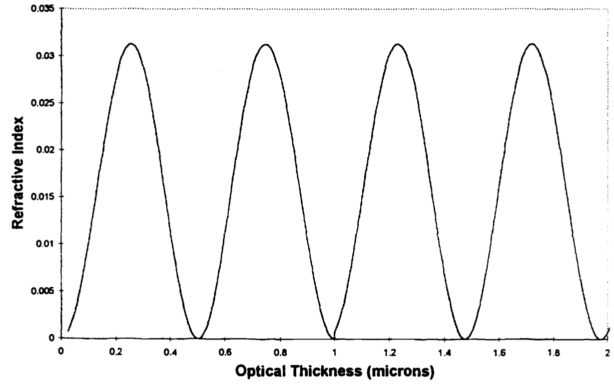

Figure 4b: The difference between the curves (sine minus exponential sine) are plotted. The difference is periodic and twice the frequency of the principle notch.

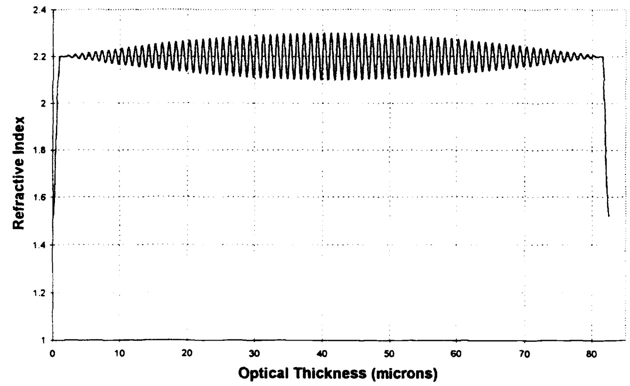

Figure 5a: Index profile for a narrower Rugate notch at 2 microns. The index excursion is 0.1. Optical thickness is 82 microns and the optical density at 2 microns is 2.65.

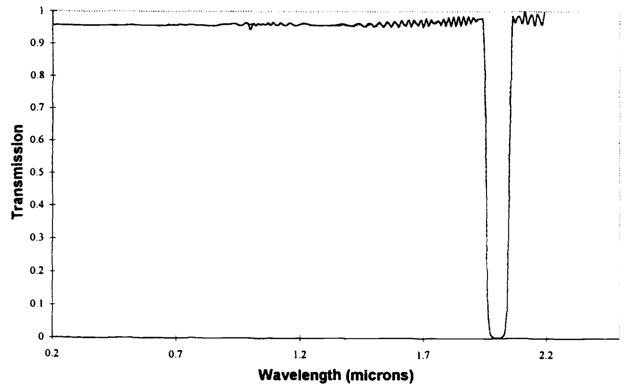

Figure 5b: Performance for sine and exponential sine wave Rugates for an index excursion of 0.1 are overlaid. The second harmonic is less noticeable at lower index excursions, and lower optical densities.

The lack of high ordered harmonics is a key design advantage making Rugate an attractive choice for broad band and hyperspectral applications. The lack of harmonics, along with index profile superposition allow the design of complex spectral performance by superimposing index profiles to get multiple reflection bands where desired. Figure set 6 presents a superposition of four index profiles to create a four notch Rugate filter. Two notches are at non-harmonic locations, and like the single notch Rugate design, lack any additional structure in the transmission spectrum. The second harmonic of the 1.064 notch was intentionally added. The bandwidth of a notch filter is a function of the index contrast, or amplitude[4]. In the case of the four notch filter example, each reflection notch has an amplitude of 0.07 requiring a total index contrast of 0.21. In designs where notch bandwidth needs to be narrow, superposition allows for efficient use of the design space by allowing the parallel rather then serial deposition of the multiple notches.

In each of these examples, transmission around the primary reflection notch is reduced by a series of digs or side-lobes. These side-lobes can be suppressed by using an apodization function to localize the wave form in much the same way apodization is used to clean-up the Fourier transform of an electronic signal. A sinusoidal apodization is illustrated in figure set 7. This technique is the sum to two index sine wave which differ by one period. The result is a profile with a single beat. Figure 7 presents an overlay of performance for the two frequencies, the unapodized notch, and the apodized notch.

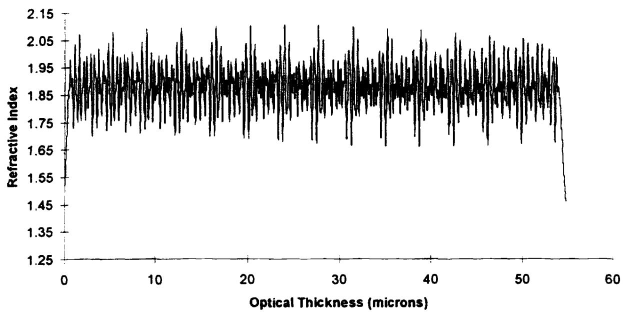

Figure 6a: Index profile for a 4 notch Rugate filter using index super position.

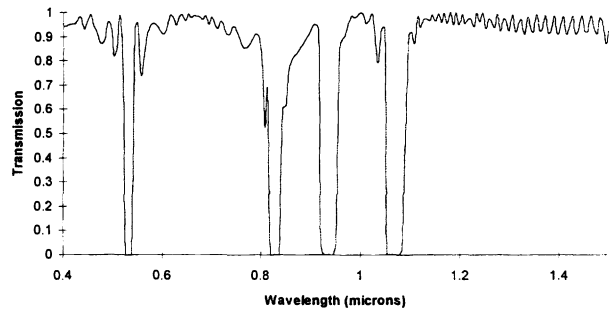

Figure 6b: Spectral performance for four notches produced in parallel at .532, .828, .935 and 1.064 microns.

Apodization is an effective means of reducing ripple around reflection notches, and can be applied to a square wave profile as well as sine wave profiles. Figure 8 through 10 present the result of applying apodization to the designs presented in figures 1 through 3. A graded index matching film is added to each design at the substrate and at the air interfaces. In each case, side-lobes and out of band ripple is reduced. Unfortunately, the growth of optical density is also severely impacted.

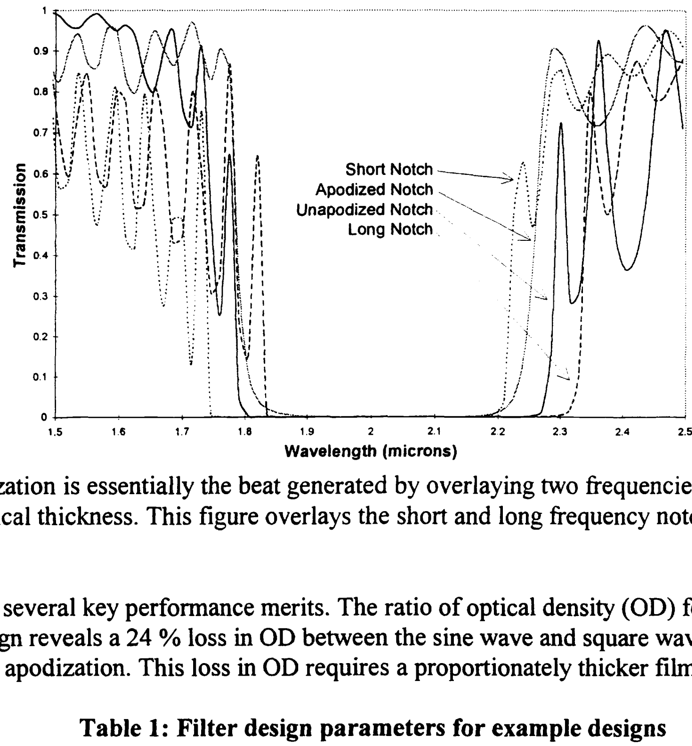

Figure 7: Sine wave apodization is essentially the beat generated by overlaying two frequencies which are 1 period different over the total optical thickness. This figure overlays the short and long frequency notches with the unapodized and apodized notch.

Table 3 lists a summary of several key performance merits. The ratio of optical density (OD) for each design to the optical density of the discrete design reveals a 24 % loss in OD between the sine wave and square wave function, and nearly a 50 % loss in optical density with apodization. This loss in OD requires a proportionately thicker film to make-up the difference.

Table 1: Filter design parameters for example designs

Parameter	Figures 1-3	Figure 5	Figures 8-10
• notch location	2.0 microns	2.0 microns	2.0 microns
• index excursion	0.36	0.1	0.36
• average index	1.86	2.1	1.86
• number of cycles or groups	20	80.5	20
• apodization	no	yes	yes
• index matching	yes	yes	yes
• high index	2.2 (TiO_2)	2.2 (TiO_2)	2.2 (TiO_2)
• low index	1.46 (SiO_2)	1.46 (SiO_2)	1.46 (SiO_2)

Table 2: Index profile equations

Waveform	Equation
Square wave	$0.5L(HL)^{19}0.5L$
Sine wave	$n_i = \left[\sum_{j=1}^{number\_of\_lines} amp_j \cdot \left(\sin\left(\frac{4 \cdot \pi \cdot OT}{\lambda_j} \right) + phase_j \right) + ave\_index \right]$
Exponential sine	$n(z) = e^{\left[0.5(\ln(nb) + \ln(na)) - (0.5(\ln(nb) - \ln(na))) \cdot \cos\left(\frac{2 \cdot \pi \cdot OT_z}{\lambda} \right) \right]}$ where: n_{ave} = average index, n_0=entrance medium, n_s = substrate index
Apodization	$\sin[\pi(\lambda_a-\lambda_b)/(2OT)]$, where: $\lambda_{notch} = (\lambda_a+\lambda_b)/2$, and $(\lambda_a-\lambda_b) = 2OT_{total}$

Table 3: Comparison of optical performance for designs in figures 1-3, and 6-9.

Waveform	OD	OD Ratio	OD Ratio	Short Edge	Long Edge	%BW (FWHM)
Square wave	5.83	1.00		1.74	2.30	28.13
Sine wave	4.44	0.76		1.81	2.23	20.63
ExpSine	4.45	0.76		1.81	2.23	20.63
Apodized Sqr	3.92	0.67	1.00	1.78	2.30	26.25
Apodized Sine	2.98	0.51	0.76	1.80	2.23	21.25
Apodized ExpSine	2.99	0.51	0.76	1.80	2.23	21.25

2.0 Filter Design Algorithms

The design of complex performance using Rugate design techniques consists of superimposing multiple index profiles. Superposition can be used to design square wave and discrete filters as well. The approach described here is to first build-up the desired performance using sine wave superposition, and then convert the wave form from a sine to a square wave to reduce film thickness, or a digitized function to allow a simpler deposition process.

A data structure reflects the underlying model of design strategy, and it can severely restrict of support design flexibility. The data structure detailed in figure 9 presents the design data for a Rugate, discrete, or filter hybrid. Design information is hierarchically ordered into filter descriptors, procedures, and lines. The filter description is high level information which describes the filter as a whole. Examples of high level data are the entrance and exit media indices. The film is made-up of procedures. The procedures can be serially linked as in the case of a matching layer, apodized Rugate notch filter, and a film to air matching procedure. Within a procedure, index profiles can be superimposed. Each profile can be a different waveform and index contrast. Procedure parameters are those which apply to all the index profiles in a specific procedure. Examples of procedure variables are average index, and materials. Line parameters are those which define the individual index profiles. Examples include number of groups or cycles, waveform, or apodization.

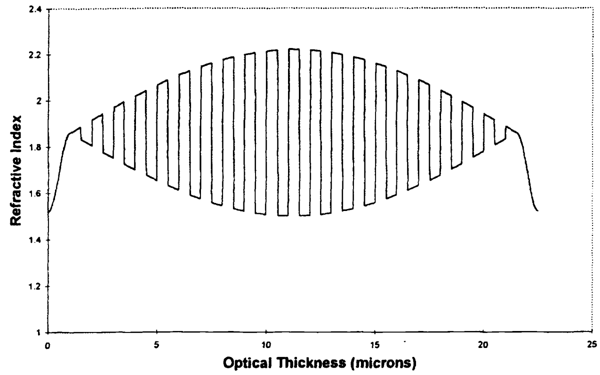

Figure 8a: Apodized gradient index square wave index profile.

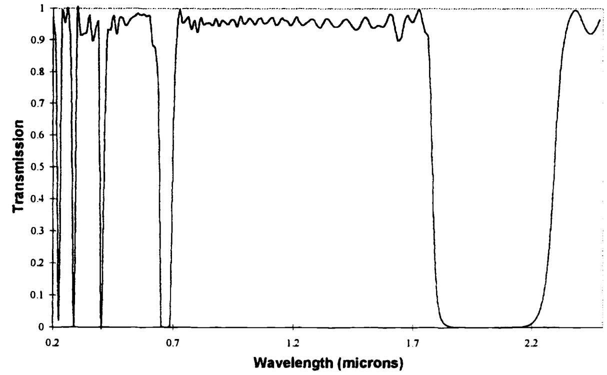

Figure 8b: Predicted performance for the apodized gradient index filter design.

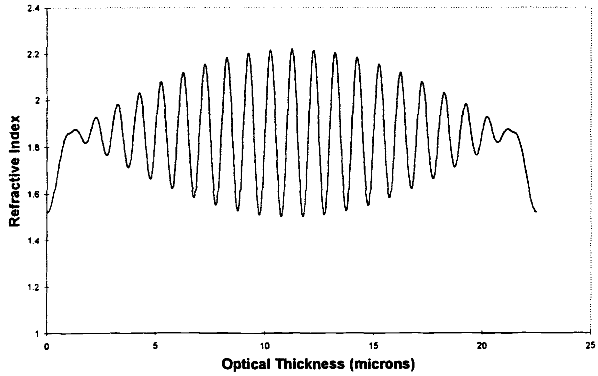

Figure 9a: Rugate sine wave index profile.

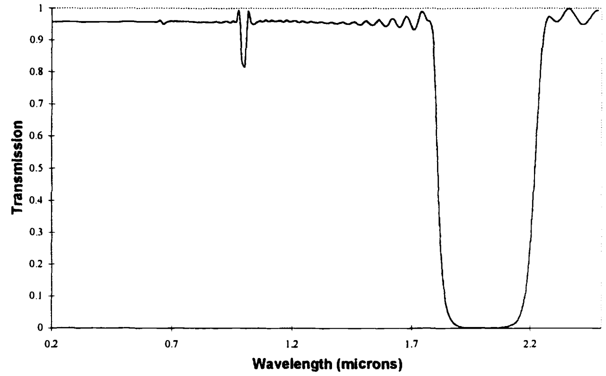

Figure 9b: Predicted performance for the Rugate, sine wave gradient index filter.

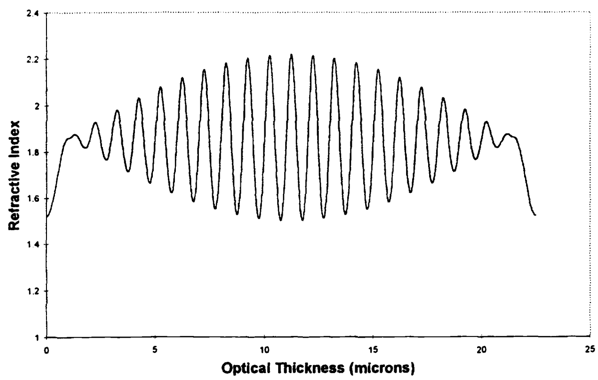

Figure 10a: Rugate exponential sine index profile.

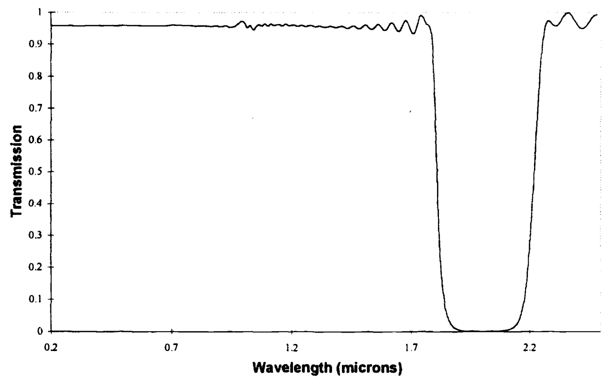

Figure 10b: Predicted performance for the Rugate, exponential sine gradient index filter.

Global Design Parameters

• Design Name	Fig 6
• Comment	
• Number of Procedures	3
• Substrate Index	1.52
• Exit Media Index	1

Procedure Variables

• Procedure Name	AR	Rugate	AR
• Procedure Type	Matching	Reflector	Matching
• Number of Lines	1	3	1
• Average Index	1.86	2.2	1.86
• Apodize	No	Yes	No
• High Index	2.3	2.3	2.3
• Low Index	1.46	1.46	1.46

Wavelength	4.0
Amplitude	.36
# Cycles	0.5
Phase	270
Functional Form	ExpSine

Wavelength	1.315	1.55	2.0
Amplitude	0.1	0.1	0.1
# Cycles	0.5	0.5	0.5
Phase	270	270	270
Functional Form	ExpSine	ExpSine	ExpSine

Wavelength	4.0
Amplitude	.36
# Cycles	0.5
Phase	270
Functional Form	ExpSine

Figure 11: Data structure for design codes. The data structure hierarchically orders the design parameters into global, procedure and line values. Procedures are deposited in series and lines within a procedure are deposited in parallel. The functional form of each line can be set individually allowing easy evaluation for the design using different functional forms. Flowcharts for convert a sine wave design to graded index square wave, frequency apodized discrete and digitized discrete designs is presented in figure set 12.

3.0 Graded Index Discrete Designs

In situations where the spectral coverage is limited to a single spectral band such as the visible spectrum, high order harmonics is not typically a concern. Changing waveform from a sine or exponential sine function to a square wave is demonstrated in figure set 13. This design is a two notch visible filter. Notch locations are 0.5 and 0.55 microns. Apodization and index matching is applied to each design. The filter is first designed using a sinusoidal index profile (figures 10a and 10b), and then converted to a square wave (figures 13c to 13f). In figure 13c and 13d, the thickness of the square wave design was reduced by 25% to more efficiently meet the desired OD of 3.0 for each notch. The band width of the square wave design is wider then the sine wave design by about the same percentage as the reduced film thickness. In figures 13e and 13f the amplitude of the index excursions is reduced to narrow up the notches, and film thickness is increased to keep the optical density of each notch at 3. In this circumstance, the film thickness is nearly the same as the sine wave profile design, but there is a more efficient use of the index contrast. More notches could be added in parallel to the square wave profile with out exceeding the index limits of the source material than with sine wave designs.

4.0 Rugates to Discrete Designs

In the previous examples, a sine wave design was converted to square wave design by changing waveform. These designs require gradient index deposition as the index can be any value within the index contrast of the material blend. Conversion to a finite number of materials allows the use of stack film technology for fabrication. Two examples of Rugate to discrete filter conversion are presented. The first example is frequency modulation rather then amplitude modulation as a means of apodizing the design. The second example is sine wave digitization onto a limited set of user specified materials as a means of suppressing high order harmonics.

Figure set 14 presents a comparison of amplitude and frequency modulated index apodization of a square wave index profile. Figure 14a and 14b are similar to earlier apodization examples. Apodization of the index amplitude tends to reduce the bandwidth and optical density of the filter design. Figures 14c and 14d use frequency modulation. Sine the total contrast of the film is not affected, the optical density and notch bandwidth is closer to the unapodized quarter wave stack. This design requires three materials, and it can be deposited as a discrete stack. The drawback of this approach is the requirement for very thin layers at the start and end of the run. Frequency modulated apodization can also be applied to sine wave index profiles with similar improvements in optical density and bandwidth.

Digitizing a sine wave index profile onto a limited matrix of discrete materials is an effective means of designing a filter with limited harmonic suppression. Figure 15a illustrates this technique. A sine wave Rugate is first designed, then a limited number of materials (typically 3 to 5) is specified. The selection and thickness of each layer is determined by a digitization algorithm working off the sine wave profile. Figures 15b ,15c, and 15d present an example of this technique using 2, 3 and 5 materials.

5.0 Summary

Waveform plays a critical role in determining filter performance. Sine wave Rugates are an effective, and intuitive means of building-up complex performance. Using Rugate design techniques as a starting point, square wave and discrete designs can be derived. Square wave, gradient index profiles can be advantageous in reducing the optical thickness of a design, or in making more efficient use of the index contrast. Discrete designs digitized from sine wave profiles provide a straightforward means of designing discrete filters with harmonic suppression.

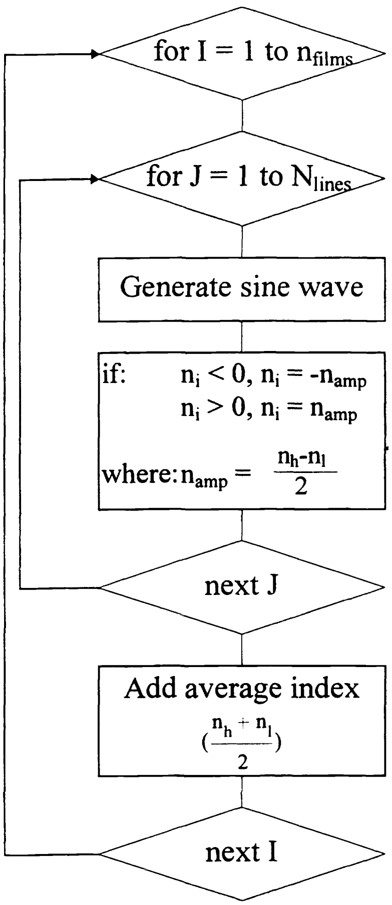

Figure 12a: Flowchart for converting from a sine index profile to a gradient index square wave profile.

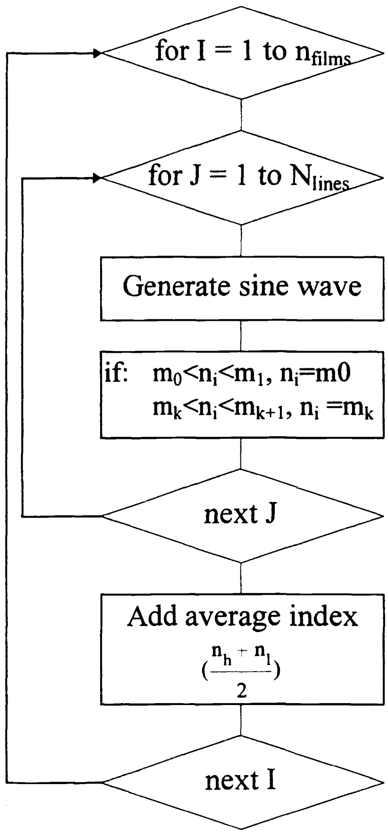

Figure 12b: Flowchart for converting from a sine wave index profile to a frequency modulated apodized discrete design.

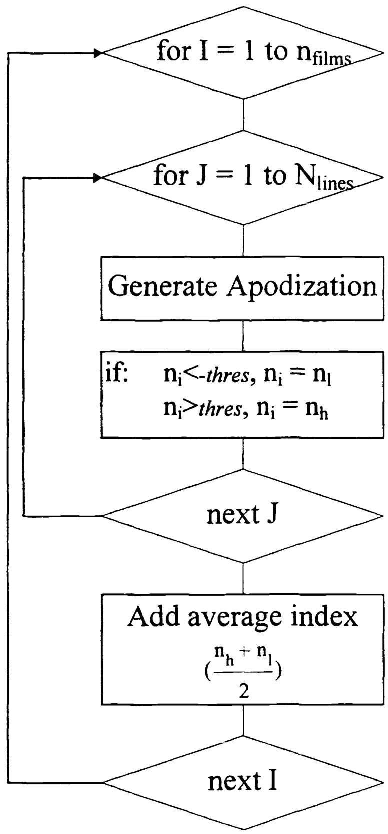

Figure 12c: Flowchart for converting from a sine wave index profile to a digitized discrete design.

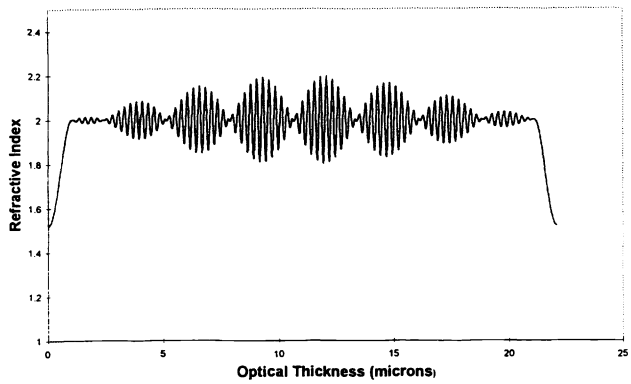

Figure 13a: Sine wave index profile for a two notch filter.

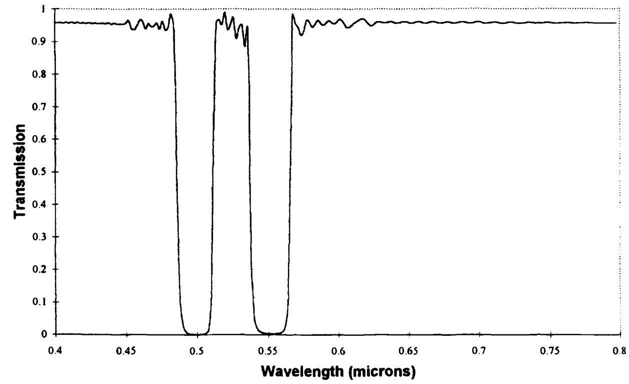

Figure 13b: Predicted performance for the two notch sine wave filter.

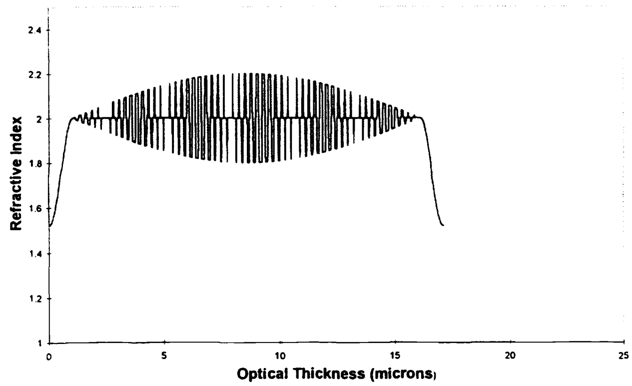

Figure 13c: Square wave index profile for a two notch filter. Film thickness is reduced by 25% to give the same optical density as the sine wave design above.

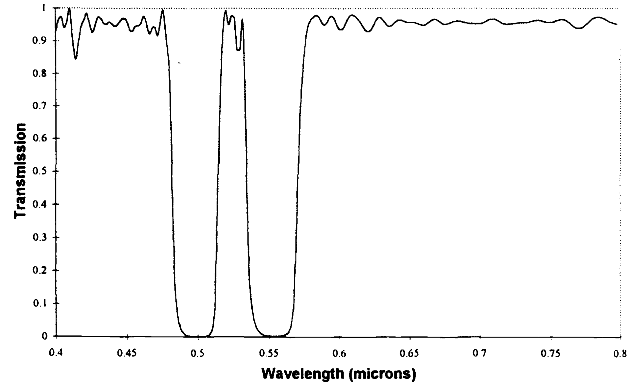

Figure 13d: Predicted performance for the two notch square wave filter. Bandwidth is about 23% wider then the sine wave design.

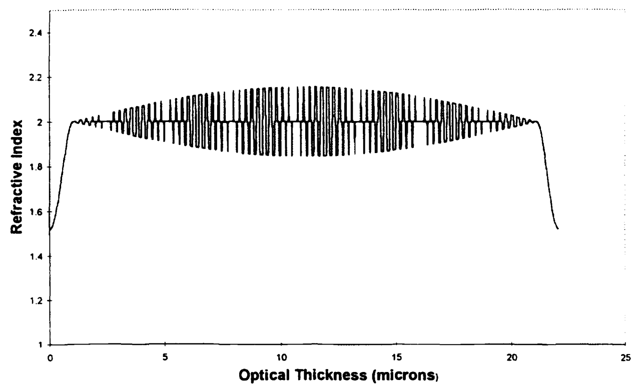

Figure 13e: Square wave index profile for a two notch filter. Index amplitude for each line is reduced by 23% to give the same bandwidth and optical density as the sine wave design in figure 10a.

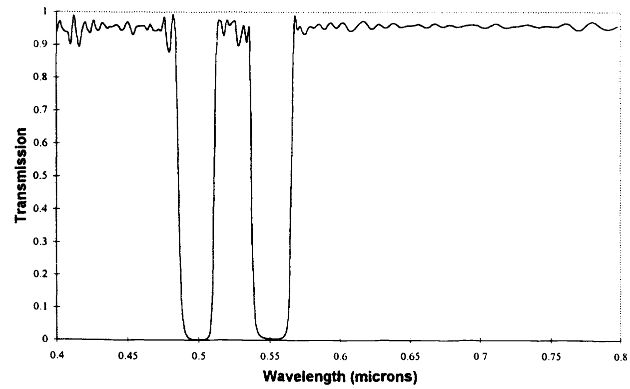

Figure 13f: Predicted performance for a two notch square wave filter. Optical thickness is the same as figure 10b, but the index amplitude is reduced by 23%. Optical density and bandwidth are now comparable to the sine wave design, but less of the index contrast is used.

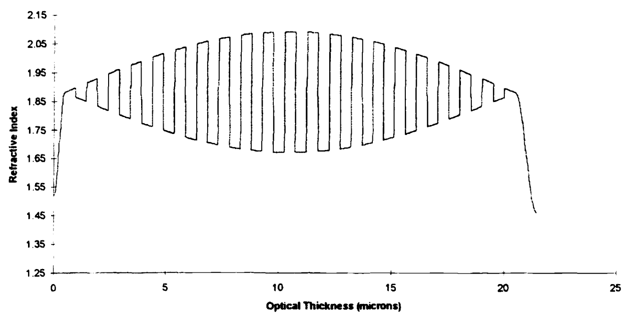

Figure 141a: Index profile for an amplitude modulated, apodized square wave filter. This design requires gradient index deposition.

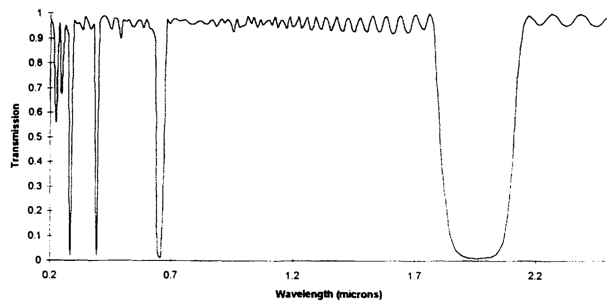

Figure 14b: The spectral performance of the amplitude modulated design exhibits reduced bandwidth and optical density which is characteristic of amplitude modulation.

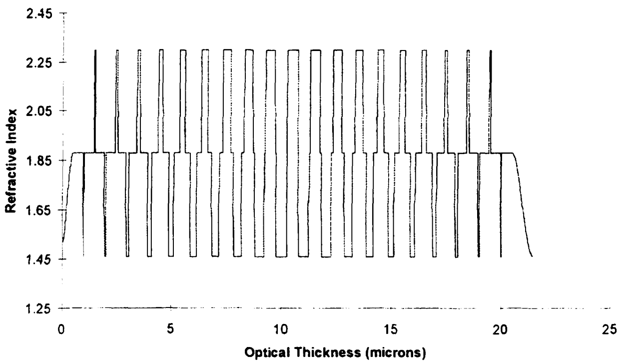

Figure 141c: Index profile for a frequency modulated, apodized square wave filter. This design consists of a three layer, discrete film system.

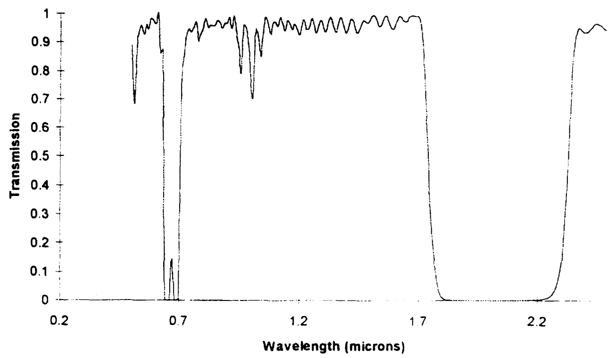

Figure 14d: Spectral performance of the frequency apodized design hold much of the bandwidth and optical density of the unapodized quarter wave stack.

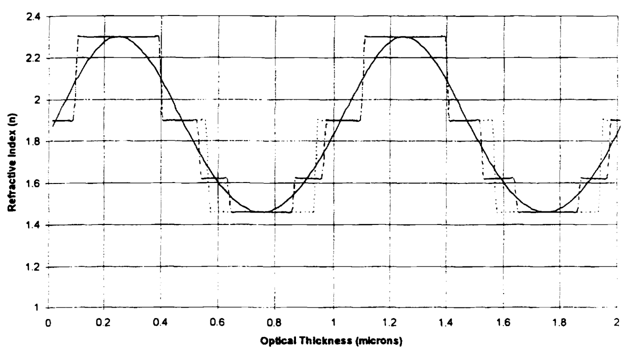

Figure 15a: A sine wave index profile can be initially designed, and digitized onto a discrete material matrix.

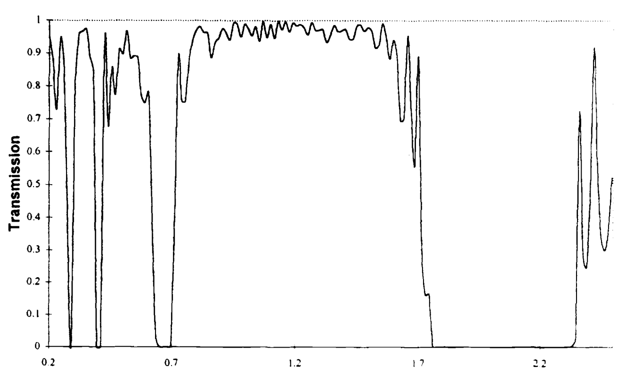

Figure 15b: Discrete, two material stack design. Materials and indices modeled are TiO_2: 2.3, and SiO_2: 1.46[5].

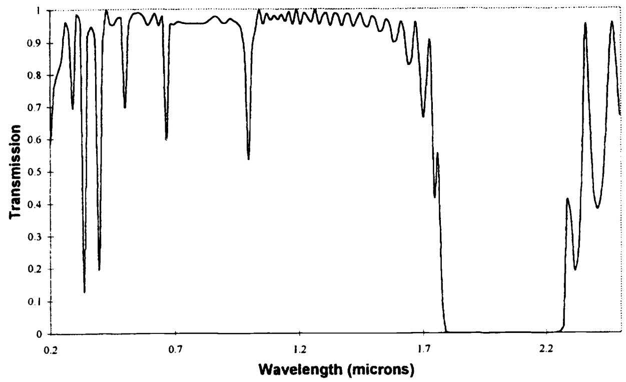

Figure 15c: Digitized three material stack design. Materials and indices modeled are TiO_2: 2.3, Al_2O_3: 1.67, and SiO_2: 1.46

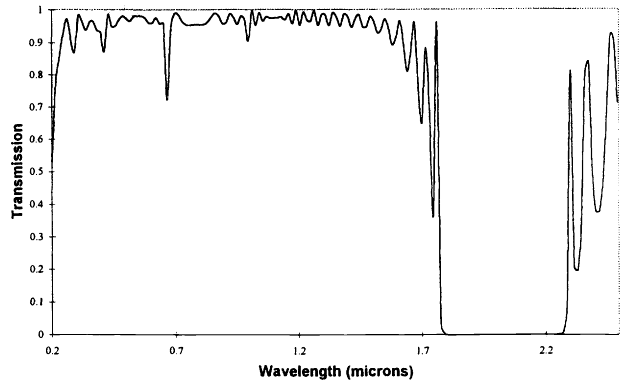

Figure 15c: Digitized five material stack design. Materials and indices modeled are TiO_2: 2.3, Ta_2O_5: 2.0, Nb_2O_5: 1.9, Al_2O_3: 1.67, and SiO_2: 1.46.

6.0 References

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